

Algebra II 6.6 Solve Exponential and log equations.

Obj: Solve exponential and log equations

Property of equality for exponents: If $b > 0$ and $b \neq 1$ then

then $b^x = b^y$ iff $x = y$
 $2^3 = 2^x$ $x = 3$

match the bases.

Example 1. Finding a common base. Solve $3^{2x+1} = 81$ (need a base the same for both)

$3^{2x+1} = 81$ base 3
 $3^{2x+1} = 3^4$
 $2x+1 = 4$ \rightarrow $2x = 3$
 $x = \frac{3}{2}$

You try. $\frac{1}{2} = 4^{3x}$ use base 2

$(2^{-1})^{x+7} = (2^2)^{3x}$
 $2^{-x-7} = 2^{6x}$
 $-x-7 = 6x$ \rightarrow $-7 = 7x$
 $-1 = x$

no calc required

$25^{3x} = 125^{x+2}$ use base 5

$(5^2)^{3x} = (5^3)^{x+2}$
 $6x = 3x+6$ $3x = 6$
 $x = 2$

Example 2. Review rewrite using exponents if no common base.

like last lesson yes you have a calculator

$17 = 4^x$

$\log 17 = \log 4^x$
 $\log 17 = x \log 4$ power prop.

$\frac{\log 17}{\log 4} = x$

IF you start w/ exp \rightarrow log or ln both to solve!

() are your friends

Example 3. Exponents on both sides. You must log both.

$$5^x = 3^{x+1}$$

Don't forget ()

$$\ln 5^x = \ln 3^{x+1}$$

$$x \ln 5 = (x+1) \ln 3$$

$$x \ln 5 = x \ln 3 + \ln 3$$

$$x \ln 5 - x \ln 3 = \ln 3$$

$$x(\ln 5 - \ln 3) = \ln 3$$

$$x = \frac{\ln 3}{\ln 5 - \ln 3} = \frac{\ln 3}{\ln \frac{5}{3}}$$

$$\approx 2.15$$

You try. $2^{3x} = 7^{x+1}$

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$$\ln 2^{3x} = \ln 7^{x+1}$$

$$3x \ln 2 = (x+1) \ln 7$$

$$3x \ln 2 = x \ln 7 + \ln 7$$

$$3x \ln 2 - x \ln 7 = \ln 7$$

$$x(3 \ln 2 - \ln 7) = \ln 7$$

$$x = \frac{\ln 7}{3 \ln 2 - \ln 7}$$

Example 4. Using an exponential model. The diagram shows how a forest fire grows over time. The fire department can contain a 160-acre fire without needing additional resources. About how many minutes does it take for a fire to become too big for the fire department to contain without additional resources? Round to the nearest minute.

$$a = 4$$

$$r = 7.2/4 \approx 1.8$$

$$y = 4(1.8)^t$$

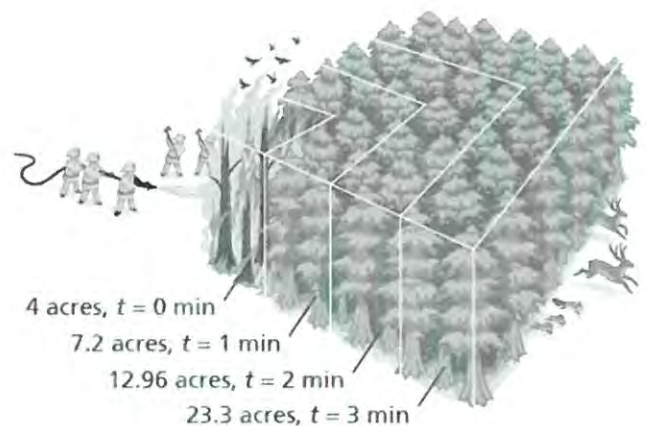
$$160 = 4(1.8)^t$$

$$40 = 1.8^t$$

$$\ln 40 = \ln 1.8^t$$

$$\ln 40 = t \ln 1.8$$

$$t = \frac{\ln 40}{\ln 1.8} \approx 6.276 \text{ min.}$$



About how many minutes does it take for a fire to spread to cover 100 acres.

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$$100 = 4(1.8)^t$$

$$25 = 1.8^t$$

$$\ln 25 = \ln 1.8^t$$

$$\ln 25 = t \ln 1.8$$

$$t = \frac{\ln 25}{\ln 1.8} \approx$$

Watch for Extraneous!

Property of equality for logs: If $x > 0$ and $b \neq 1$ then

$$\log_b x = \log_b y \quad x = y$$

Example 5. Solve. $\ln(x^2 - 16) = \ln(6x)$

$$\begin{aligned} x^2 - 16 &= 6x \\ x^2 - 6x - 16 &= 0 \\ (x - 8)(x + 2) &= 0 \\ x = 8 \quad x &= \cancel{-2} \end{aligned}$$

$$\begin{aligned} \ln(6x - 16) &= \ln 48 \\ \ln 48 &= \ln 48 \checkmark \end{aligned}$$

$$\ln(4 - 16) = \ln(-12)$$

\times can't \ln a negative

logs on both side

combine then drop logs

Watch for extraneous solutions!

$x = 8$ is the only solution!

Solve using properties. Combine to one log.

$$3 \log_5 x - \log_5 4 = \log_5 16$$

$$\log_5 x^3 - \log_5 4 = \log_5 16$$

$$\log_5 \frac{x^3}{4} = \log_5 16$$

$$\frac{x^3}{4} = 16$$

$$\begin{aligned} x^3 &= 64 \\ x &= 4 \checkmark \end{aligned}$$

Solve when logs on one side only. Combine and rewrite.

$$\log_4 x + \log_4 (x - 6) = 2$$

$$\log_4 (x^2 - 6x) = 2$$

$$x^2 - 6x = 16$$

$$x^2 - 6x - 16 = 0$$

$$(x - 8)(x + 2) = 0$$

$$x = 8 \quad x = \cancel{-2}$$

Check on GC with intersection.

$$\log_8 x + \log_8 (x - 12) = 2$$

$$\log_8 (x^2 - 12x) = 2$$

$$x^2 - 12x = 64$$

$$x^2 - 12x - 64 = 0$$

$$(x - 16)(x + 4) = 0$$

$$x = 16 \quad x = \cancel{-4}$$

You cannot just drop the logs!

If log start w/ logs on one side
Do the opposite \rightarrow use exponents to solve!

